



The calculation methods for each of the operations should be **well modelled, practised and applied in a variety of problem solving activities at each level**. This will ensure the children have depth of understanding, rather than being moved to quickly to the next level. It is equally important not to allow children to ‘stick’ at a level when they should be moving on. Children must be secure in concrete and pictorial methods before moving onto abstract. KS1 must focus on understand number and basic operations before moving onto any kind of written method.

## Addition Methods

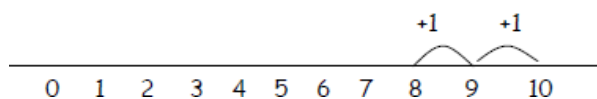
KS1	LKS2	UKS2																																															
To count and add together sets of real objects and pictures	To and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	To add and subtract whole numbers with up to 5 digits, including using formal written methods																																															
<ul style="list-style-type: none"> <li>Children should handle, group and add real objects and pictures, and count aloud.</li> </ul> <p>3+2 = 5</p>  <p>OR</p> 	<ul style="list-style-type: none"> <li>Extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.</li> </ul> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right;">42</td> <td style="text-align: right;">37</td> <td style="text-align: right;">68</td> </tr> <tr> <td style="text-align: right;">23</td> <td style="text-align: right;">12</td> <td style="text-align: right;">35</td> </tr> <tr> <td style="text-align: right;"><u>14</u></td> <td style="text-align: right;"><u>26</u></td> <td style="text-align: right;"><u>14</u></td> </tr> <tr> <td style="text-align: right;"><u>89</u></td> <td style="text-align: right;"><u>75</u></td> <td style="text-align: right;"><u>117</u></td> </tr> <tr> <td></td> <td style="text-align: right;">1</td> <td style="text-align: right;">11</td> </tr> </table> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right;">243</td> <td style="text-align: right;">364</td> </tr> <tr> <td style="text-align: right;"><u>146</u></td> <td style="text-align: right;"><u>237</u></td> </tr> <tr> <td style="text-align: right;"><u>389</u></td> <td style="text-align: right;"><u>601</u></td> </tr> <tr> <td></td> <td style="text-align: right;">11</td> </tr> </table>	42	37	68	23	12	35	<u>14</u>	<u>26</u>	<u>14</u>	<u>89</u>	<u>75</u>	<u>117</u>		1	11	243	364	<u>146</u>	<u>237</u>	<u>389</u>	<u>601</u>		11	<ul style="list-style-type: none"> <li>Extend to adding numbers up to 5 digits and numbers with numbers with different numbers of digits.</li> </ul> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right;">52243</td> <td style="text-align: right;">64364</td> </tr> <tr> <td style="text-align: right;"><u>11156</u></td> <td style="text-align: right;"><u>31237</u></td> </tr> <tr> <td style="text-align: right;"><u>63399</u></td> <td style="text-align: right;"><u>95601</u></td> </tr> <tr> <td></td> <td style="text-align: right;">11</td> </tr> </table> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right;">32865</td> <td></td> </tr> <tr> <td style="text-align: right;"><u>28327</u></td> <td></td> </tr> <tr> <td style="text-align: right;"><u>61192</u></td> <td></td> </tr> <tr> <td style="text-align: right;">11 1</td> <td></td> </tr> </table> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right;">62431</td> <td style="text-align: right;">32855</td> </tr> <tr> <td style="text-align: right;">14324</td> <td style="text-align: right;">28327</td> </tr> <tr> <td style="text-align: right;"><u>21134</u></td> <td style="text-align: right;"><u>25413</u></td> </tr> <tr> <td style="text-align: right;"><u>97889</u></td> <td style="text-align: right;"><u>86595</u></td> </tr> </table>	52243	64364	<u>11156</u>	<u>31237</u>	<u>63399</u>	<u>95601</u>		11	32865		<u>28327</u>		<u>61192</u>		11 1		62431	32855	14324	28327	<u>21134</u>	<u>25413</u>	<u>97889</u>	<u>86595</u>
42	37	68																																															
23	12	35																																															
<u>14</u>	<u>26</u>	<u>14</u>																																															
<u>89</u>	<u>75</u>	<u>117</u>																																															
	1	11																																															
243	364																																																
<u>146</u>	<u>237</u>																																																
<u>389</u>	<u>601</u>																																																
	11																																																
52243	64364																																																
<u>11156</u>	<u>31237</u>																																																
<u>63399</u>	<u>95601</u>																																																
	11																																																
32865																																																	
<u>28327</u>																																																	
<u>61192</u>																																																	
11 1																																																	
62431	32855																																																
14324	28327																																																
<u>21134</u>	<u>25413</u>																																																
<u>97889</u>	<u>86595</u>																																																

To add one or several more onto a number line

To add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

- Children should have lots of practice of counting along a number line, and then using it to add one extra, and then several.

$$8 + 2 = 10$$

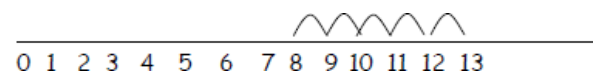


- Extend to adding numbers up to 4 digits and numbers with numbers with different numbers of digits.

$$\begin{array}{r} 2243 \\ 1156 \\ \hline 3499 \end{array} \qquad \begin{array}{r} 4364 \\ 1237 \\ \hline 5601 \\ 11 \end{array}$$

To be able to add through 10, some children bridging through 10

- Children should practise adding one and then several, so that the answer is ten and then beyond.



To know that addition can be done in any order.

Use knowledge by starting with biggest number.

- Children should practise adding numbers in any order and be allowed to discover that numbers can be added in any order to reach the same answer.
- Children should be taught to begin with the most significant number and add smaller ones on.
- Children should be taught to spot number bonds to ten and add those first.

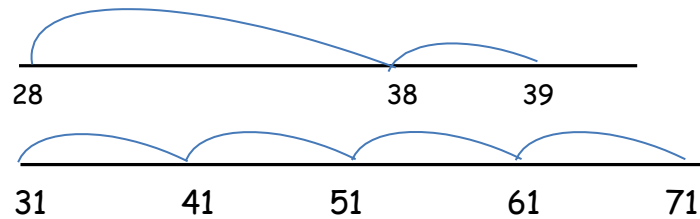
$$\begin{aligned}3+9 &= 9+3 \\ &= 12\end{aligned}$$

$$\begin{aligned}3+7+2 &= 7+3+2 \\ &= 10+2 \\ &= 12\end{aligned}$$

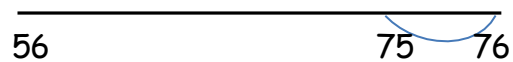
To be able to add 10 to any number up to 100.  
 To be able to add multiples of 10 to any number up to 100. To be able to add 11 or 21 to a 2 digit number up to 100. To be able to add 9 or 19 to a 2 digit number by adding 10 or 20 and subtracting 1.

Children should be supported in their understanding by using a number line, 100 square or other appropriate apparatus.

- Children add 10 and then 1 or 20 then 1 to a 2 digit number.
- Children add 10 and then subtract 1 or 20 then subtract 1. to a 2 digit number
- Children should be given plenty of practice leading to them being able to calculate mentally.



56+19



$45+11 = 56$

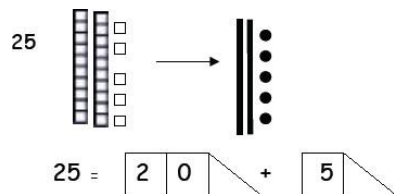
24	25	26	27	28
34	35	36	37	38
44	45	46	47	48
54	55	56	57	58
64	65	66	67	68

$45+11 = 45+10+1$   
 $= 55+1$   
 $= 56$

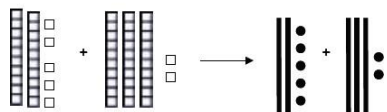
$45+21 = 45+20+1$   
 $= 65+1$   
 $= 66$

To be able to partition 2-digit numbers  
 To be able to add two 2-digit numbers using practical partitioning

- Children will **partition** 2-digit numbers by practical partitioning into tens and ones/units, then drawings to represent, and arrow cards.
- Children **add** two 2-digit numbers by practical partitioning into tens and ones/units, then drawings to represent.



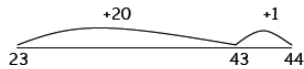
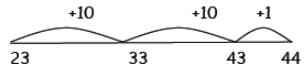
$25 + 32 = 57$



To be able to add two 2-digit numbers on an empty number line.

- Children place one of the two 2-digit numbers onto an empty number line and add the second by jumps of ten, and then by jumps of ones.
- Extend to adding multiples of ten and jumps of units to the next decade number.

$$23 + 21 = 44$$



$$48 + 36 = 84$$

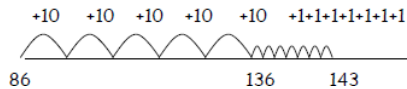


To be able to add two 2-digit numbers which sum to more than 100 on an empty number line

To be able to add two 2-digit numbers which sum to more than 100 by partitioning

- Children place the larger of two 2-digit numbers onto an empty number line and add the second by jumps of ten, and then by jumps of ones

$$86 + 57 = 143$$



- Children partition both 2-digit numbers into their respective tens and ones/units, add each part separately, and recombine to find the answer.


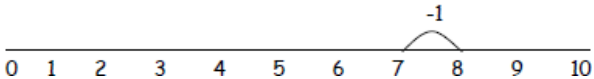
To add numbers using formal column method

- In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry one ten' or 'carry one hundred', not 'carry one'.
- Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.

$$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ 11 \end{array} \quad \begin{array}{r} 258 \\ + 87 \\ \hline 345 \\ 11 \end{array} \quad \begin{array}{r} 366 \\ + 458 \\ \hline 824 \\ 11 \end{array}$$

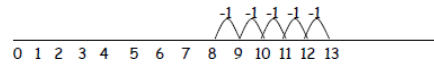
Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

## Subtraction Methods

<p>To be able to take away real objects</p> <ul style="list-style-type: none"> <li>Children should handle and practise taking away real objects and pictures.</li> </ul> <p><math>5 - 2 = 3</math></p> 		
<p>To be able to subtract/take away one less on a number line</p> <ul style="list-style-type: none"> <li>Children should have lots of practice of placing a number on a number line, and then using it to find one less/subtract/take away by jumping one place back.</li> </ul> <p><math>8 - 1 = 7</math></p> 	<p>To subtract using Column subtraction</p> <ul style="list-style-type: none"> <li>Recording is reduced further. When decomposition is necessary the altered numbers are shown above the actual numbers.</li> <li>Extend to subtracting two-digit from three-digit and numbers with different numbers of digits.</li> </ul> <p>1)      2)      3)      4)</p> $\begin{array}{r} 38 \\ -12 \\ \hline 26 \end{array}$ $\begin{array}{r} \overset{5}{\cancel{3}}3 \\ -15 \\ \hline 48 \end{array}$ $\begin{array}{r} \overset{4}{\cancel{3}}\overset{11}{2}3 \\ -59 \\ \hline 464 \end{array}$ $\begin{array}{r} \overset{7}{\cancel{3}}\overset{9}{\cancel{0}} \\ -43 \\ \hline 757 \end{array}$	
<p>To be able to subtract through 10, some children bridging through 10.</p>		

- Children should have lots of practice of placing a number greater than 10 on a number line and counting back, subtracting one at a time.

$$13 - 5 = 8$$

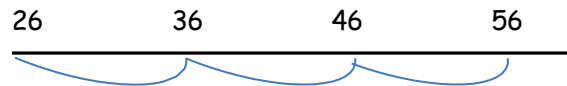


To be able to subtract 10 from any number up to 100 To be able to subtract multiples of 10 from any number up to 100

To be able to subtract 11 or 21 to a 2 digit number up to 100

- Children should subtract 10 from 2-digit numbers using a 100-square or number line. Children could confirm their understanding by counting on to check their result.
- To subtract 11 or 21 - Children should locate the higher number and first subtract 10 then the ones. Children could confirm their understanding by counting on to check their result.
- Eventually this will be done mentally

$$56 - 32 = 24$$

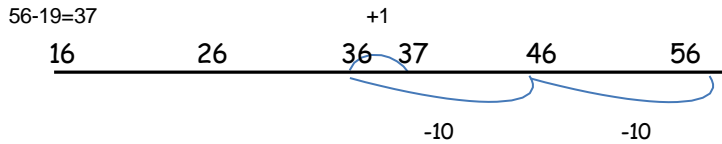


$$34 - 10 = 24$$

2	3	4	5	6
12	13	14	15	16
22	23	24	25	26
32	33	34	35	36
42	43	44	45	46
52	53	54	55	56

To be able to subtract 9 or 19 from a 2-digit number by subtracting 10 or 20 and adding 1

- Children should locate the higher number then subtract 10 or 20 then add one. Children could confirm their understanding by counting on to check their result.
- Eventually this will be done mentally

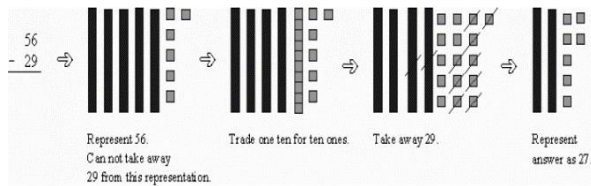
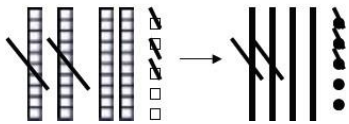


This can also be carried out using a 100sq.

To be able to subtract two 2-digit numbers by practical partitioning into tens and ones

- Children will partition 2-digit numbers by practical partitioning into tens and ones using Base 10 equipment, then drawings to represent.
- Extend to practical partitioning and subtraction where exchange of a 10 for ten ones is necessary.

45 - 23 = 22

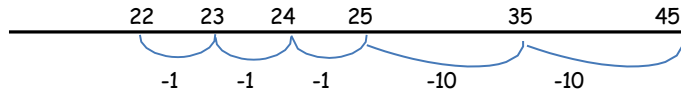


$$\begin{array}{r} 56 \\ - 29 \\ \hline \end{array} \Rightarrow \begin{array}{r} (50 + 6) \\ - (20 + 9) \\ \hline \end{array} \Rightarrow \begin{array}{r} (40 + 16) \\ - (20 + 9) \\ \hline (20 + 7) = 27 \end{array}$$

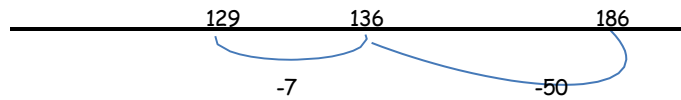
## To be able to subtract two 2-digit numbers by counting back on an empty number line

- Children place the higher of the two 2-digit numbers onto an empty number line and subtract the second by jumps of ten, and then by jumps of ones.
- Extend to subtract a 2-digit numbers from a 3-digit number

$$45 - 23 = 22$$



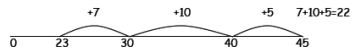
$$186 - 57 = 129$$



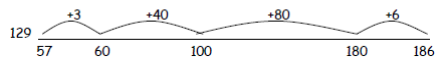
## To be able to subtract two 2-digit numbers, finding the difference by counting up on an empty number line

- Children place the number to be subtracted onto an empty number line and count up to the higher number by jumping to the next decade number, then in tens, then in a jump to the unit of the higher number.
- Children should record systematically
- Extend to adding two 2-digit to 3-digit number
- Extend to adding decimals where no more than three columns would be needed.

$$45 - 23 = 22$$



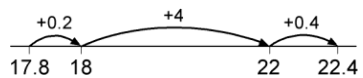
$$186 - 57 = 129$$



$$40 + 80 = 120$$

$$3 + 6 = 9$$

$$120 + 9 = 129$$


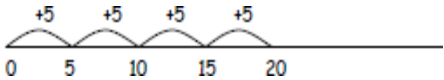


## To be able to subtract two 2-digit numbers using partitioning

- Recording is reduced further. When decomposition is necessary the altered numbers are shown above the actual numbers.
- Extend to subtracting two-digit from three-digit and numbers with different numbers of digits.

1)	2)	3)	4)
38	<sup>5</sup> <del>3</del> 3	<sup>4</sup> <sup>11</sup> <del>2</del> 3	<sup>7</sup> <sup>9</sup> <del>0</del> 0
-12	-15	- 59	- 43
----- 26	----- 48	----- 464	----- 757

## Multiplication Methods

KS1	LKS2	UKS2
<p>To be able to practically count repeated groups/sets of the same size.            To count mentally in 2s, 5s and 10s.            To use objects to work out repeated addition</p>	<p>To multiply using the short multiplication method</p>	<p>To find products of two 2-digit numbers</p>
<p>Children should have lots of practice of addition of the same number of objects (or the numbers themselves) repeatedly. Children to use objects or draw.</p> <p><math>2 + 2 + 2 + 2 + 2 = 10</math></p> 		
<p>To relate repeated addition to multiplication.</p>		
<p>Introduce multiplication symbol. Children should practise recording repeated addition and relate to multiplication.            Children need to know how this is represented on a number line.</p> <p><math>5 + 5 + 5 + 5 = 20</math>  <math>4 \times 5 = 20</math></p> 		

**To multiply using arrays.**

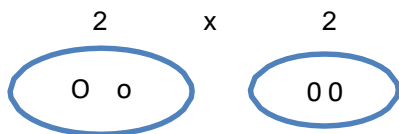
Children should be able to display multiplication as an array and be able to discuss that this shows 4 lots of 5, or 5 lots of 4.

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



OR



**To multiply TU by U using short multiplication**

The recording is reduced further, with carry digits recorded below the line.

If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.

$$\begin{array}{r} 38 \\ \times 7 \\ \hline \end{array}$$

**To find products of two 2-digit numbers**

- Reduce the recording further.
- The carry digits in the partial products of  $56 \times 20 = 120$  and  $56 \times 7 = 392$  are usually carried mentally.

The aim is for most children to use this long multiplication method for TU  $\times$  TU by the end of Year 5.

$56 \times 27$  is approximately  $60 \times 30 = 1800$ .

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \quad 56 \times 7 \\ 1120 \quad 56 \times 20 \\ \hline 1512 \\ 1 \end{array}$$

**To count mentally in 3s and 4s**

To learn 2, 5, and 10 times tables and division facts by heart

To practise multiplication problems with missing numbers

Children need lots of oral practice and are able to record on paper.

Begin to introduce corresponding division facts

$$3 \times 5 = ?$$

$$3 \times ? = 15$$

$$? \times 5 = 15$$

### To build families of number facts

Children to learn remaining tables and build families of number facts.

To introduce inverse

$$3 \times 5 = ? \qquad ? \div 3 = 5$$

$$3 \times ? = 15 \qquad 15 \div 3 = ?$$

$$? \times 5 = 15 \qquad 15 \div 5 = ?$$

### To find products of 3-digit and 2-digit numbers



Children who are already secure with multiplication for  $TU \times U$  and  $TU \times TU$  should have little difficulty in using the same method for  $HTU \times TU$ .

Again, the carry digits in the partial products are usually carried mentally.

$286 \times 29$  is approximately  $300 \times 30 = 9000$ .

$$\begin{array}{r} 286 \\ \times 29 \\ \hline 2574 \quad 286 \times 9 \\ 5720 \quad 286 \times 20 \\ \hline 8294 \\ 1 \end{array}$$

# Division Methods

KS1	LKS2	UKS2
<p>To be able to half numbers to 20-practically and eventually mentally.</p> <p>To be able to divide practically by sharing.</p>	<p>To practise division problems with missing numbers</p>	<p>To divide HTU by TU, using 'Expanded' method, with no remainder</p>
<p>Children should have plenty of practice, practically sharing a variety of objects between 2, and then higher numbers of people.</p> <p>6 eggs shared between 2 nests = 3</p> 	<p>Children need lots of oral practice and to be able to record on paper.</p> <p><math>? \div 3 = 5</math></p> <p><math>15 \div 3 = ?</math></p> <p><math>15 \div 5 = ?</math></p>	<p>'Short' division of HTU <math>\div</math> U can be introduced as an alternative, more compact recording</p> <p>For most children this will be at the end of Year 5 or the beginning of Year 6.</p> $\begin{array}{r} \underline{033} \quad 15 \\ 15) \underline{49}45 \\ \quad 45 \quad 4 \\ \quad \quad 49 \end{array}$
<p>To be able to divide practically by grouping.</p>		<p>To divide HTU by TU, using 'Expanded' method, with a remainder</p>
<p>Children should have plenty of practice making different groupings, starting with a small number and increasing to larger numbers of objects.</p> <p>6 eggs put into groups of 2 = 3</p> 		$\begin{array}{r} \underline{042} \text{ r}2 \quad 15 \\ 15) \underline{63}32 \\ \quad 60 \quad 3 \\ \quad \quad 63 \end{array}$

To relate sharing and grouping to division symbol	To carry out short division of TU ÷ U	To divide HTU by U, using 'Expanded' method, including decimals
<p>Children should have plenty of practice relating their practical experience to the written representation, using the division symbol.</p> <p><b>6 eggs put into groups of 2 = 3</b>  <b>6 ÷ 2 = 3</b></p>	<p>'Short' division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning.</p> <p>Short division of a two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.</p> <p>For most children this will be at the end of Year 4 or the beginning of Year 5.</p> <p>This is then shortened to:</p> $\begin{array}{r} 27 \\ 3 \overline{) 821} \end{array}$	<div style="display: flex; justify-content: space-around;"> <div style="text-align: left;"> <math display="block">\begin{array}{r} 04.2 \text{ r}2 \\ 15 \overline{) 63.32} \\ \underline{63} \phantom{.} \\ \phantom{63} \phantom{.} 32 \end{array}</math> </div> <div style="text-align: left;"> <math display="block">\begin{array}{r} 15 \\ 30 \\ 45 \\ 60 \quad \overset{3}{\curvearrowright} \\ \phantom{60} \phantom{\curvearrowright} 63 \end{array}</math> </div> </div> <div style="display: flex; justify-content: space-around;"> <div style="text-align: left;"> <math display="block">\begin{array}{r} 041.6 \text{ r}2 \\ 12 \overline{) 491.74} \\ \underline{48} \phantom{.} \\ \phantom{48} \phantom{.} 19 \phantom{.} \\ \phantom{48} \phantom{.} 19 \phantom{.} 74 \end{array}</math> </div> <div style="text-align: left;"> <math display="block">\begin{array}{r} 12 \quad \overset{7}{\curvearrowright} \\ 24 \quad \phantom{\curvearrowright} 19 \\ 36 \\ 48 \quad \overset{1}{\curvearrowright} \\ 60 \quad \phantom{\curvearrowright} 49 \\ 72 \quad \overset{2}{\curvearrowright} \\ \phantom{72} \phantom{\curvearrowright} 74 \end{array}</math> </div> </div> <p>In Year 6 answers are to be given as a fraction or decimal rather than remainders.</p>

### To relate division to multiplication using arrays

Children should be able to recognise arrays as being the physical representations of multiplication facts, and be able to verbalise them. For example,  $18 \div 6 = 3$  could be discussed as 18 shared into 6 groups, or the question could be asked "How many lots/groups of 3 in 18?"

$$6 \times 3 = 18$$



$$3 \times 6 = 18$$

$$18 \div 6 = 3$$

$$18 \div 3 = 6$$

### To carry out short division

Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

$$\begin{array}{r} 97 \\ \underline{3)2921} \\ 15 \\ 18 \\ 21 \\ 24 \\ 27 \\ 30 \\ 33 \\ 36 \end{array}$$

$\begin{array}{r} 7 \\ \hline 17 \overline{)123456.} \\ -119 \\ \hline 44 \end{array}$	$\begin{array}{r} 72 \\ \hline 17 \overline{)123456.} \\ -119 \\ \hline 44 \\ -34 \\ \hline 105 \end{array}$	$\begin{array}{r} 726 \\ \hline 17 \overline{)123456.} \\ -119 \\ \hline 44 \\ -34 \\ \hline 105 \\ -102 \\ \hline 36 \end{array}$
$\begin{array}{r} 7262.1 \\ \hline 17 \overline{)123456.0} \\ -119 \\ \hline 44 \\ -34 \\ \hline 105 \\ -102 \\ \hline 36 \\ -34 \\ \hline 20 \end{array}$	$\begin{array}{r} 7262.11... \\ \hline 17 \overline{)123456.00} \\ -119 \\ \hline 44 \\ -34 \\ \hline 105 \\ -102 \\ \hline 36 \\ -34 \\ \hline 20 \\ -17 \\ \hline 30 \end{array}$	